Agitation and Mixing
Basic Theoretical Concepts

Stir and Mix
Stirring and mixing are complex homogenization operations involving hydrodynamic (discharge regime), thermal (transfers), chemical (reaction) and mechanical (shearing) phenomena. These operations, when carried out industrially, require the implementation of efficient mixing systems to ensure the stability and the consistency of the mixtures at low costs (minimal time and energy). It is within this context that blenders by mechanical rotation were imposed in many industries for all kinds of mixtures:

<table>
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<tr>
<th>Mobilization</th>
<th>Miscible liquids</th>
<th>Homogenization</th>
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<tbody>
<tr>
<td>Inmiscible liquids</td>
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<td>Suspension</td>
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<td>Dissociation</td>
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<th>Dispersion (phase rupture)</th>
<th>Inmiscible liquids</th>
<th>Homogenization</th>
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<tr>
<td>Liquids - Gas</td>
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<td>Dispersion - Emulsion</td>
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Agitation parameters
Advocating and proportioning a type of blenders is to determine the optimum parameters for the implementation of the sought after process. This optimization is very often done under constraints, whether of costs, congestion or physical limitations. This approach is based on the choice of a number of parameters:

Agitator type(s) and positioning:
- Radial flow blades
- Axial flow blades
- Mixed flow blades
- Tangential flow blades
- Dispersion / emulsification blades
- Bowl geometry (dimensions, forms)
- Blade rotation (speed, discharge regime)
- Mixture duration
- Physical conditions imposed (pressure, temperature)

Agitation energy
These elements are predominant in calculating the main characterization parameter of the mixing system that is the dissipated energy (or necessary energy for its driving), which allows:
- selecting the engine to install
- comparing mixture performances
- diagnose and possibly pilot the mixture operation

\[ P = 2 \pi N (C - C_0) \]

\( P \) in Vacuum Couple (N.m)
\( W \) in (tr/s)
\( N \) in s\(^{-1}\)
\( C \) in liters
\( C_0 \) in liters

This energy is based on:
- the nature of the mixture: viscosity (\( \mu \) in Pa.s), density (\( \rho \) in Kg/m\(^3\))
- the blender type: geometries and dimensions (where \( d \), agitation tool diameter in m)
- the operating mode: rotation speed (\( N \) in s\(^{-1}\)), acceleration of gravity (\( g \) in m/s\(^2\))
Non-dimensional number characteristics
Each of these values which can be expressed from the three fundamental units (volume, length, time), the Vaschy-Buckingham theorem can transform the expression of energy, as a first approach, into 3 non-dimensional numbers linked together:

- **Reynolds number**:
  \[ R_e = \frac{(N d)^2 \rho}{\mu} \]
  which characterizes the relationship between the inertial forces and the viscous forces. \( R_e \) allows calculating the type of discharge flow (laminar or turbulent; \( R_e \) elevated → turbulent regime)

- **Froude number**:
  \[ F_r = \frac{N^2 d}{g} \]
  which characterizes the relationship between inertial forces and the gravity forces. \( F_r \) allows predicting the formation of a vortex (\( F_r \) elevated → important vortex)

- **Power number**:
  \[ N_p = \frac{P}{\rho N^4 d^4} \]
  which is the drag coefficient of the agitator in the fluid and hence the power consumption expression

In laminar regime (\( R_e < 10 \)): \( N_p R_e = \text{Cte} = A \)
\[ \mu \rho \frac{dNRe}{dNRe} = \text{constant} \]
\( \mu \rho \frac{dNFr}{dNFr} = \text{constant} \]

In turbulent regime (\( R_e > 10^4 \)): \( N_p = \text{Cte} \)
\[ \mu \rho \frac{dN}{dN} = \text{constant} \]
\( \mu \rho \frac{dN}{dN} = \text{constant} \)

Other parameters can better characterize the flow within the mixer:
- The pumping number (non-dimensional) : \( N_{np} \rightarrow \) flow of material which passes through the agitation blade
- The circulation number (non-dimensional) : \( N_{qc} \rightarrow \) flow of material which circulates around the agitation blade
- The shearing rate (in s⁻¹): \( \gamma \)

Mixing criteria
All these parameters allow:
1. To calculate, for a given geometry, the necessary power to obtain a mixture and hence the total energy consumed by the agitator,
2. To calculate, for a given energy, the dimensions of the agitator,
3. To estimate mixing times \( t_m \) which are themselves linked to circulation times \( t_c \) (average time taken by a material element to finish a complete rotation in the bowl), by applying the rule \( N_p t_c = \text{Cte} \)
4. To determine if it is better to run faster or longer in time (while respecting energy constraints)
5. To interpolate the characteristics of a certain type of mixers to a mixture of different dimensions
Thermal transfers in the agitated bowls
The agitated bowls are rarely used for the sole purpose of heating or cooling of the material, because they are less effective than heat exchangers designed for this function. However, it is necessary to bring or to evacuate the heat inside the bowls during mixing operations.

These problems of heat transfer are of particular concern when trying to interpolate the characteristics of a certain bowl to a bowl of different dimensions.

The calculations of heat transfers in a bowl are governed by the laws of thermodynamics and especially by the law of exchange through a partition between two environments of different temperatures:

\[
\frac{dQ}{dt} = \frac{\Theta_c - \Theta_i}{h_{c}dA_{c} + h_{f}dA_{f}}
\]

where :
- \( h_{c} \) and \( h_{f} \) are the surface coefficients of heat transfer
- \( dA_{c} \) and \( dA_{f} \) are corresponding surfaces
- \( \lambda_{t} \) is the thermal conductivity of the solid material (partition)
- \( dA_{m} \) is the logarithmic mean of \( dA_{c} \) and \( dA_{f} \) areas

The surface coefficients of heat transfer that can be obtained by the expression of the following non-dimensional numbers:
- the Nusselt number which expresses the relationship between the transferred flow and that which will be transferred if the fluid was at rest
  \[
  Nu = \frac{h_{i}D}{\lambda_{i}}
  \]
  where \( D \) is the bowl diameter
- the Prandtl number which can be considered as the ratio of heat diffusion and movement quantity coefficients
  \[
  Pr = \frac{C_{pi}\mu_{i}}{\lambda_{i}}
  \]
- the viscosity number which is the ratio between the mean viscosity of the fluid in the bowl (\( \mu \)) and that calculated at the partition temperature (\( \mu_{p} \))
  \[
  V_{i} = \frac{\mu}{\mu_{p}}
  \]

In order to obtain the thermal exchange coefficients, we generally use the correlation which consists of writing \( Nu \) in the form:

\[
Nu = k.R_{c}C^{\alpha}Pr^{\beta}V_{i}^{\gamma}
\]

To Learn More:

Literature
- Agitation et Mélange - Catherine Xuereb, Martine Poux, Joël Bertrand (Editions Dunod)
- Mixing : Principles and Applications - S. Nagata (Editions John Wiley)
- Dossier Agitation, Mélange -Michel Roustan, Jean-Claude Pharamond (Techniques de l’Ingénieur)